CHAPTER 1

INTRODUCTION

There are different aspects of the supersymmetric potential, here we have used the perturbation process which is proposed by P. B. Abraham and H. E. Moses [1]. They have worked on a solved problem of quantum mechanical one dimensional system. After that they have constructed an algorithm to find the exact perturbation which gave the exact form of the potential for supersymmetric partner. By using Gelfand-Levitan method [2] they have shown the elimination of the ground state energy for the potential of a system where the other bound state energy has not changed at all.

M. Bernstein and L. S. Brown [3] have shown some properties of supersymmetric potential which is remarkable. Using Fokker-Planck equation [4], they have shown the bistable potential property for "bosonic" particle and single potential well for the "fermionic" particle.

C. V. Sukumar [5] has presented a systematic procedure for constructing a hierarchy of non-relativistic Hamiltonians. This is the procedure we have followed to find the potential where the eigenvalue spectrum remained the same except the ground state eigenvalue. He has also developed the potentials predicted by Nieto [6].

M. M. Nieto [7] has explained how the eigenvalue spectrum will be changed for the perturbation. He has shown that (i) the normalization of the eigenvectors, having the same eigenvalue, are changed or (ii) a finite number of eigenvalue are subtracted from the spectrum or (iii) a finite number of eigenvalue are added to the spectrum.

[Fig 1: Schematic diagram of possible eigenvalue spectrum where (i) eliminating ground state, (ii) adding ground state and (iii) normalization of eigenvector has been shown]

D. Baye [8] has shown that the the deep and shallow nature of nucleus-nucleus potential can be describe by supersymmetric quantum mechanics. He has also shown that the shallow potentials are singular and strongly angular momentum quantum number dependent and supersymmetric shallow potentials are phase equivalent or phase differed by an integral multiple of π.

D. Baye [9] has also shown that a coupled-channel potential can be determined which is phase equivalent to a given potential and whose bound spectrum is identical except for one arbitrary bound state which is removed.

F. Cooper, A. Khare and U. Sukhatme [10] has shown the theoretical formulation of supersymmetric quantum mechanics and many application of supersymmetry. They have shown the property of shape invariance for all solvable potential.

A. A. Andrianov, M. V. Ioffe and V.P.Spiridonov [11] has shown how the Witten index [12] is changed for higher order supersymmetry.

C.V. Sukumar [13] has constructed reflectionless potential and phase equivalent potential using supersymmetry quantum mechanics.

Using variational method in supersymmetry quantum mechanics, E. D. Filho and R. M. Ricotta [14] have constructed the energies of both the Harmonic Oscillator and the Hulthen potential confined in three dimensions.

V. A. Kosteleck´ and N. Russell [15] have applied supersymmetric quantum mechanics to describe the particles trap. They have shown that the supersymmetric-partner wave functions can be used to describe a valence fermion in a trap system with an isotropic harmonic-oscillator potential.

A. Gangopadhyaya, J. V. Mallow, C. Rasinariu and U. P. Sukhatme [16] has shown a process by which one can obtain analytic expressions for the eigenvalues and eigenfunctions for all nonrelativistic shape invariant Hamiltonians.

A. Sergyeyev and B. M. Szablikowski [17] have constructed the cotangent universal hierarchy. After that they have constructed (2+1)-dimensional double extension of the cotangent universal hierarchy.

M. M. Nieto [18] has shown how a non-analytical potential cab be developed.

M. S. Berger and V. A. Kostelecky [19] have constructed supersymmetric field theories that violate Lorentz and CPT symmetry. They have illustrated this with some examples related to the original Wess-Zumino model.[20]

D. J. Fernandez C and N. F. Garcia [21] have reviewed the higher-order supersymmetric quantum mechanics which involves differential intertwinning operators of order greater than one. They have used iterations of first-order SUSY transformations and direct technique for second order transformations.

In this paper, we have discussed about the characteristics of the potential of a supersymmetric partner with the eigenvalue spectrum. We have calculated such potential using the well known potential system of Simple Harmonic Oscillator potential and Square well potential. We have calculated the Schrödinger equation involving supersymmetric partner of Simple Harmonic Oscillator and Square well potential both analytically and numerically when necessary.

CHAPTER 2

FORMALISM

2.1. Supersymmetric Algebra

There are four fundamental interactions [22] in the nature. They are weak interaction, strong interaction, electromagnetic interaction and gravitational interaction. The physicists are trying to unify all the interactions for a long time.The electromagnetic interaction and weak interaction are already unified into electroweak interaction and also electromagnetic interaction and strong interaction into electrostrong interaction [23]. But total unification still remains unfinished. The term supersymmetry is introduced in string theory [10] to the unification process. It is thought that using this in quantum field theory, all the fundamental interaction can be unified.

The term symmetry means the system remains invariant under translation or rotations and hence the momentum become a constant of motion [24]. It can be represented as

$$[H, p_x] = 0 \tag{2.1.1}$$

this commutator relation means that the Hamiltonian H is symmetric under translations and the momentum p_x is a constant of motion.

Supersymmetry is a symmetry relating bosonic and fermionic degrees of freedom [25]. This is an exciting idea but the implementation is complicated. We have seen that the symmetry contains commutator relation. But anti-commutator relations are involve in the supersymmetrc algebra [26] as

$$\{Q_i, Q_j\} = \delta_{ij} H \tag{2.1.2}$$

$$\{Q_i, H\} = 0 \tag{2.1.3}$$

where Q_i are the charge operator.

In terms of particle states, a supermultiplet contains just two types of particles, differing by a $\frac{1}{2}$ unit of helicity. Heisenberg [27] has shown that perturbation theory break down completely at high energy (about 300GeV) for the higher order effects in the theory of weak interaction. To explain this he have explored the idea of SUSY in 1970. Though the evidence of Supersymmetry [28] has not directly observed, in nuclear physics experimental evidence of the presence of supersymmetry is found in nature.

It invites us to contemplate in fermionic dimension i.e., the degrees of freedom can extend the space-time co-ordinates. We can say that SUSY enlarge space-time to "superspace" [10].

The phenomenologically important and true aspects is that SUSY implies degenerate multiplets of bosons and fermions [29].

It not only works well in Standard Model (SM) of particle physics but also provide a solution to the "hierarchy problem" [10]. The another more important remarks that we have from Supersymmetry is that it enables the String Theory.

For these various reason there are some "try it and see" approches to constructing SUSY invariant theories.

SUSY algebra was first described by Haag, Lopuszanski and Sohnius [30]. Here algebra means the generator of the appropriate symmetry transformations i.e., a group of transformations that leaves the Lagrangian invariant [31] and must obey the conservation law which is Noether's theorem [32]. The essential feature is that the superpartner have anticommutation relations among themselves. Thus the SUSY algebra is involved with some commutation relations and some anticommutation relations.

A fundamental aspect of any symmetry is the algebra associated with the symmetry generators [25]. The generators T_i of supersymmetry satisfy the commutation relations

$$[T_i, T_j] = i\epsilon_{ijk} T_k \tag{2.1.4}$$

where i, j and k run over the values $1, 2$ and 3, and where the repeated index k is summed over i.e., $\epsilon_{123} = +1$, $\epsilon_{213} = -1$ etc. Which is the same as angular momentum operators in quantum mechanics, in the unit $\hbar = 1$.

If the charge operator Q_i be the generator then we have

$$\{Q_i, Q_j\} = \delta_{ij} H \tag{2.1.5}$$

$$\{Q_i, H\} = 0 \tag{2.1.6}$$

Where H is the Supersymmetric Hamiltonian and charge operators can represented as

$$Q = \begin{bmatrix} 0 & 0 \\ A^- & 0 \end{bmatrix} \qquad Q^\dagger = \begin{bmatrix} 0 & A^+ \\ 0 & 0 \end{bmatrix} \tag{2.1.7}$$

There is no direct transformations between fields with different integer spins [33] i.e., the charge operator, Q_i can only change the spin by $\frac{1}{2}$ and

$$H = \{Q, Q^\dagger\} = \begin{bmatrix} A^+A^- & 0 \\ 0 & A^-A^+ \end{bmatrix} \tag{2.1.8}$$

From these algebric relations we have found some basic property of supersymmetry [26]. The properties are:

- All particles belonging to one supermultiplet have the same mass.
- In a supersymmetric theory the energy H is always positive or semi-positive.
- A supermultiplet always contains an equal number of bosonic and fermionic degrees of freedom.

2.2. Factorization of Hamiltonian

Let us consider a Hermitian positive semi-definite operator of the form $H = A^+A^-$ in which A^+ is the Hermitian adjoint of the operator A^-. Let ψ be an eigenfunction of H with eigenvalue E. The eigenvalue equation

$$A^+A^-\psi = E\psi \tag{2.2.1}$$

leads, on multiplication from the left be A^-, to

$$A^-A^+(A^-\psi) = E(A^-\psi) \tag{2.2.2}$$

Equations (1)and (2)lead to the following theorem.

THEOREM 2.2.1. *An eigenvalue of the operator A^+A^- is also an eigenvalue of the operator A^-A^+, except when $A^-\psi = 0$. The normalised eigenfunctions of $A^+A^-$and A^-A^+, denoted by $\psi$and ϕrespectively, are connected by the equations*

$$\phi = E^{-\frac{1}{2}}A^-\psi, \qquad \psi = E^{-\frac{1}{2}}A^+\phi \tag{2.2.3}$$

Bernstein and Brown [3] consider a Hamiltonian of the form $H_+ = A^+A^-$, with $A^{\pm} = (\mp\frac{\partial}{\partial x} + \frac{1}{2}\frac{\partial U}{\partial x})$ for a specified function $U(x)$. They showed that the scalar Hamiltonian H_+ and its 'partner' $H_- = A^-A^+$, corresponding to the potentials $V_{\pm} = (\frac{1}{2}\frac{\partial U}{\partial x})^2 \pm \frac{1}{2}\frac{\partial^2 U}{\partial x^2}$, can be viewed as the 'bosonic' and the 'fermionic' components of a supersymmetric Hamiltonian [10] as

$$H = [(-\frac{\partial^2}{\partial x^2} + W^2)I + \sigma_3\frac{\partial W}{\partial x}] \tag{2.2.4}$$

in which $W = -\frac{1}{2}\frac{\partial U}{\partial x}$, I is the unit matrix and σ_3 is the Pauli spin matrix. Since H_+ has a ground state with eigenvalue $E = 0$ and an eigenfunction that satisfies $A^-\psi = 0$, theorem 1 implies the following mapping of the eigenvalues of H_- and H_+:

$$E_-^{(n)} = E_+^{(n+1)} \qquad n = 0, 1, 2\ldots\ldots\ldots \tag{2.2.5}$$

Thus, the energy of the first excited state of H_+ by calculating the ground state energy of H_-.

We consider the non-relativistic Hamiltonian $H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x)$ for any potential $V(x)$ that can support at least one bound state. We factorise H in the form

$$H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x) \equiv A^+A^- + \varepsilon \tag{2.2.6}$$

where

$$(2.2.7) \qquad A^{\pm} = \frac{1}{\sqrt{2}}(\pm\frac{\partial}{\partial x} + \tilde{V}(x))$$

The unknown function $\tilde{V}(x)$ and the undetermined constant ε are then determined by the consistency condition that

$$(2.2.8) \qquad \tilde{V}^2 + \frac{\partial \tilde{V}}{\partial x} = 2(V - \varepsilon)$$

This condition is clearly satisfied if

$$(2.2.9) \qquad \tilde{V} = \frac{1}{\psi^{(0)}}\frac{\partial \psi^{(0)}}{\partial x} \qquad \& \qquad \varepsilon = E^{(0)}$$

where $\psi^{(0)}$ and $E^{(0)}$ are the groundstate eigenfunction and eigenvalue of H. The choice of the wavefunction in equation (2.2.9) is motivated by the consideration that A^+A^- is required to be a positive semi-definite operator with eigenvalues ≥ 0. This leads to the following theorem.

THEOREM 2.2.2. *Any Hamiltonian of the form* $H = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x)$,*which has a ground state* $(\psi^{(0)}, E^{(0)})$ *can be factorised as* $H = A^+A^- + E^{(0)}$ *with* $A^{\pm} = \frac{1}{\sqrt{2}}[\pm\frac{\partial}{\partial x} + \frac{1}{\psi^{(0)}}\frac{\partial\psi^{(0)}}{\partial x}]$.

We now show that theorems 1 and 2 enable the generation of a heirarchy of the different members of the gierarchy. Starting from a Hamiltonian H_1 for a potential $V_1(x)$ that can support M bound states with a ground state $(\psi_1^{(0)}, E_1^{(0)})$ and applying theorem 2 we get

$$(2.2.10) \qquad H_1 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_1(x) \equiv A_1^+A_1^- + E_1^{(0)}$$

$$(2.2.11) \qquad A_1^{\pm} = \frac{1}{\sqrt{2}}[\pm\frac{\partial}{\partial x} + \frac{1}{\psi_1^{(0)}}\frac{\partial\psi_1^{(0)}}{\partial x}]$$

We can now construct a 'supersymmetric partner' H_2 with potential $V_2(x)$ given by

(2.2.12)
$$H_2 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_2(x) \equiv A_1^- A_1^+ + E_1^{(0)}$$

(2.2.13)
$$V_2(x) = V_1(x) + [A_1^-, A_1^+] = V_1(x) - \frac{\partial^2}{\partial x^2} ln\psi_1^{(0)}$$

Since $A_1^-\psi^{(0)} = 0$, theorem 1 then shows that the spectra of $H_1$and H_2 satisfy the condition that

(2.2.14)
$$E_2^{(n)} = E_1^{(n+1)} \qquad n = 0, 1, 2, ...(M-2)$$

and the normalised eigenfunctions of H_1 and H_2 are connected by the equation

(2.2.15)
$$\psi_2^{(n)} = [E_1^{(n+1)} - E_1^{(0)}]^{-\frac{1}{2}} A_1^- \psi_1^{(n+1)}$$

By applying theorem 2 to the new Hamiltonian H_2 we can refactorise H_2in terms of its ground state $(\psi_2^{(0)}, E_2^{(0)})$ as

(2.2.16)
$$H_2 = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_2(x) \equiv A_2^+ A_2^- + E_2^{(0)}$$

(2.2.17)
$$A_2^{\pm} = \frac{1}{\sqrt{2}}[\pm\frac{\partial}{\partial x} + \frac{1}{\psi_2^{(0)}}\frac{\partial \psi_2^{(0)}}{\partial x}]$$

This is the new factorisation of H_2 in turn leads to a new 'supersymmetric partner' H_3 given by

(2.2.18)
$$H_3 = A_2^- A_2^+ + E_2^{(0)}$$

whose spectrum can be determined by the application of theorem 1.

By reapeted application of theorems 1 and 2 we can thus generate a hierarchy of Hamiltonians given by

$$H_n = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V_n(x) \equiv A_n^+ A_n^- + E_n^{(0)} = A_{n-1}^- A_{n-1}^+ + E_{n-1}^{(0)} \tag{2.2.19}$$

$$A_n^{\pm} = \frac{1}{\sqrt{2}}[\pm\frac{\partial}{\partial x} + \frac{1}{\psi_n^{(0)}}\frac{\partial \psi_n^{(0)}}{\partial x}] \tag{2.2.20}$$

(2.2.21)

$$V_n(x) = V_{n-1}(x) - \frac{\partial^2}{\partial x^2} ln\psi_{n-1}^{(0)} = V_1(x) - \frac{\partial^2}{\partial x^2} ln(\psi_1^{(0)}\psi_2^{(0)}...........\psi_{n-1}^{(0)} \qquad n = 2,3,4.....M)$$

whose spectra satisfy the conditions

(2.2.22)

$$E_n^{(m)} = E_{n-1}^{(m+1)} = ... = E_1^{(n+m-1)}, \;\; m = 0,1,2.......M-n \;\; \& \;\; n = 2,3,4........M$$

$$\psi_n^{(m)} = [E_1^{(n+m-1)} - E_1^{(n-1)}][E_1^{(n+m-1)} - E_1^{(n-2)}]...........[E_1^{(n+m-1)} - E_1^{(0)}]^{-\frac{1}{2}}$$

$$\times A_{n-1}^- A_{n-2}^- ....A_1^- \psi_1^{(n+m-1)} \tag{2.2.23}$$

in which A_{n-1}^- is the annihilation operator of the ground state in the potential $V_{n-1}(x)$ [6].

CHAPTER 3

CALCULATIONS

3.1. Simple Harmonic Oscillator Potential and Wavefunctions

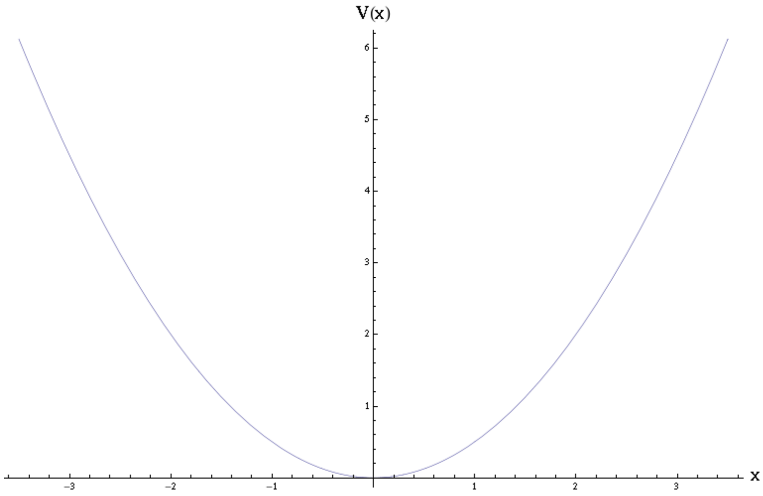
Classically, the Simple Harmonic Oscillator (SHO) is a system subject to a "restoring force" that is linear in displacement from the system's equilibrium point. If we set $x = 0$ at that point, then the force is given by: $F = -kx$, where k is a positive constant. Hence, the potential energy is $V = \frac{1}{2}kx^2$. The usual physical example of a classical SHO is a mass, m, attached to a spring. Such a system oscillates with a characteristic frequency, denoted ω. The parameters m, ω and k are related by: $\omega = \sqrt{\frac{k}{m}}$.

In quantum mechanics, the SHO can be solved through "standard" means we form the classical Hamiltonian, and then quantize it, rewriting it in terms of operators.

$$H\psi = E\psi \tag{3.1.1}$$

The time independent Schroedinger Equation can be solved exactly for this system, but we'll leave the technical aspects aside, focusing instead on results [34]. Thus in the case of SHO, we have the Potential as

$$V(x) = \frac{1}{2}m\omega^2 x^2 \tag{3.1.2}$$



[Fig 2.1.1: Simple Harmonic Oscillator Potential]

for the unperturbed system whose Hamiltonian H_0 is taken to be

$$H_0 = \frac{1}{2}\left(-\frac{d^2}{dx^2} + x^2\right) \tag{3.1.3}$$

with $\omega = 1$ and $m = 1$. Then the solution becomes the well-known eigenfunctions $\psi_n(x)$ and Eigenvalues E_n, which are

$$\psi_n(x) = \frac{1}{(\sqrt{\pi}2^n n!)^{\frac{1}{2}}} e^{\frac{x^2}{2}} H_n(x) \tag{3.1.4}$$

$$E_n = n + \frac{1}{2}, \qquad C_n = 1 \tag{3.1.5}$$

where $H_n(x)$ is the Hermite polynomials and the eigenvalue, E_n, has the unit of $\hbar\omega$ [1].

Here we see the first property of the harmonic oscillator - the allowed energy levels are equally spaced, separated by an amount $\hbar\omega$, where ω is the classical oscillation frequency. There is also a "zero point energy" - the first allowed state is not at zero energy, but instead here at $\frac{\hbar\omega}{2}$ compared to the classical minimum energy.

The first few Hemite polynomials are as follows:

$$H_0 = 1 \tag{3.1.6}$$

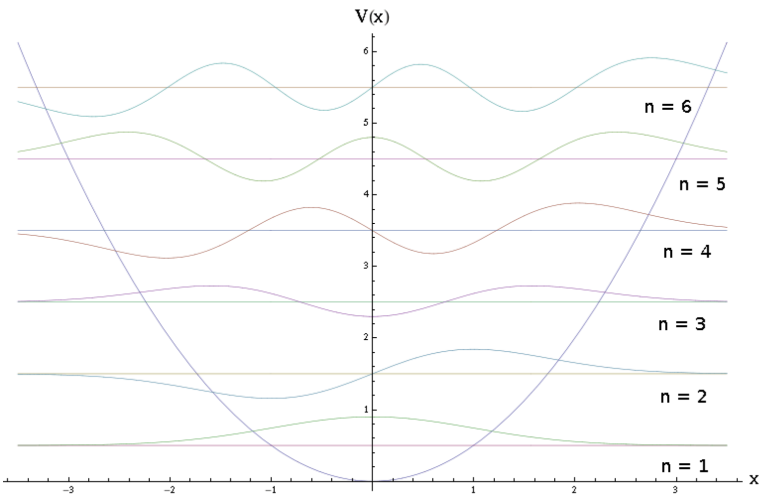
$$H_1(x) = 2x \tag{3.1.7}$$

$$H_2(x) = 4x^2 - 2 \tag{3.1.8}$$

$$H_3(x) = 8x^3 - 12x \tag{3.1.9}$$

$$H_4(x) = 16x^4 - 48x^2 + 12 \tag{3.1.10}$$

Thus the wavefunction for SHO potential with corresponding eigenvalue can be found as



[Fig 2.1.2: Wavefunctions for SHO potential]

3.2. Supersymmetric Partner of Simple Harmonic Oscillator

We have the Hamiltonian hierarchy of the superpartner and the form of potential of the supersymmetric partner. We consider the potential of the SHO as

$$V_1 = \frac{1}{2}m\omega^2 x^2 \tag{3.2.1}$$

Then we have the Schroedinger Equation as

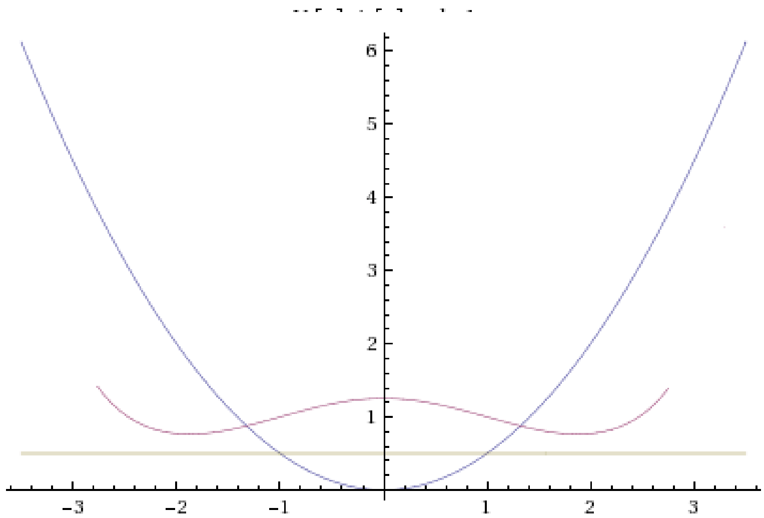
$$H\psi_n = E_n\psi_n$$
$$\implies -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_1(x) + \frac{1}{2}m\omega^2x^2\psi_1(x) = (n+\frac{1}{2})\psi_1(x)$$

$$\implies \frac{d^2}{dx^2}\psi_1(x) + \frac{2m}{\hbar^2}\left\{\left(n+\frac{1}{2}\right) - \frac{1}{2}m\omega^2x^2\right\}\psi_1(x) = 0 \tag{3.2.2}$$

We have the solution of this equation for ground state as

$$\psi_1^{(0)}(x) \sim \frac{1}{2}e^{-\frac{\omega x^2}{2}}\left(2C_1 + \sqrt{\pi}C_2 erf(x)\right) \tag{3.2.3}$$

Here we have taken $m = \hbar = \omega = 1$.



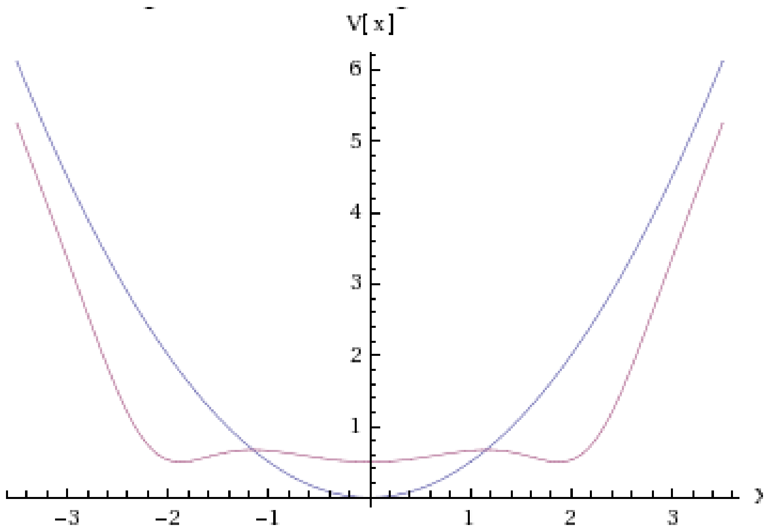
[Fig 2.2.1: SHO potential and ground state Wavefunction with eigenvalue]

Thus, the potential of the superpartner can be find out by

$$V_2(x) = V_1 - \frac{d^2}{dx^2}\ln\psi_1^{(0)}$$

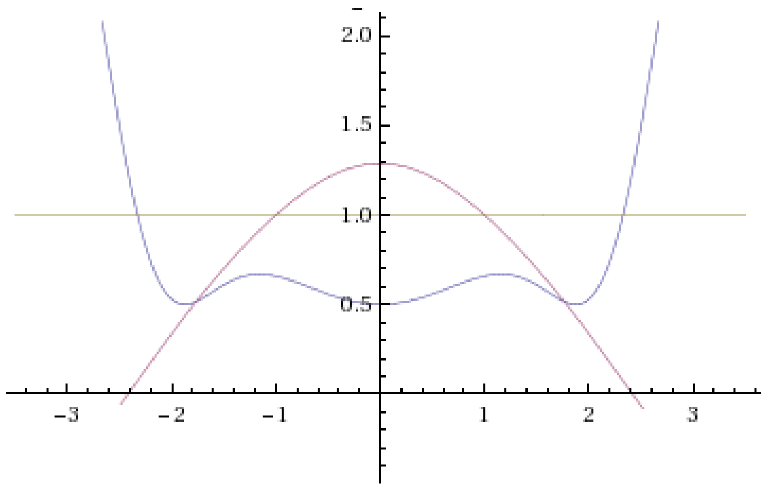
$$(3.2.4) \quad \Longrightarrow V_2 = 1 + \frac{4e^{2x^2}C_2^2}{\left(2C_1+\sqrt{\pi}C_2erf(x)\right)^2} + \frac{1}{2}x\left(x - \frac{8e^{x^2}C_2}{2C_1+\sqrt{\pi}C_2erf(x)}\right)$$

The potential V_1 and it's superpartner V_2 is shown in the following graph

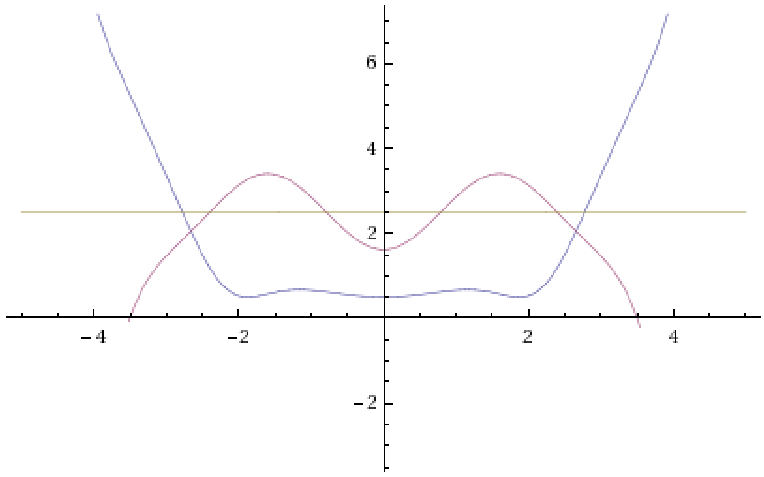


[Fig 2.2.2: The potential $V_1(x)$ and it's superpartner $V_2(x)$]

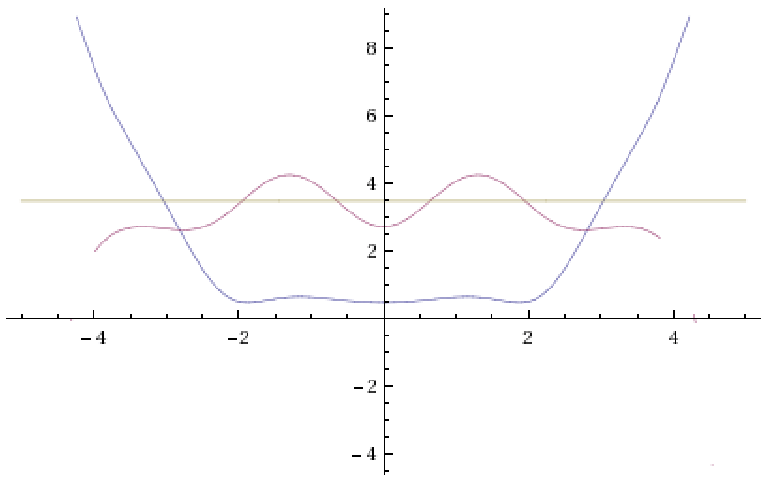
The wavefunctions $\psi_2^{(m)}$ for the potential V_2 we can get by numerically solving the Schroedinger equation and these are given below.



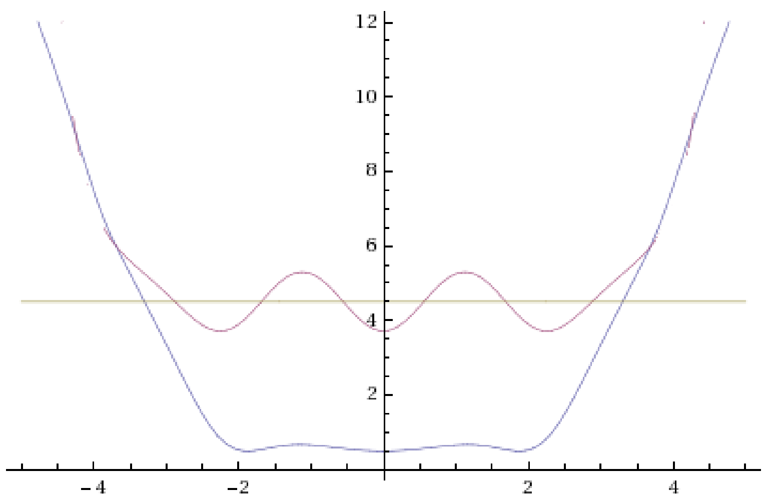
[Fig 2.2.3: $V_2(x), \psi_2^{(0)}(x)$ and $E_2^{(0)}$]



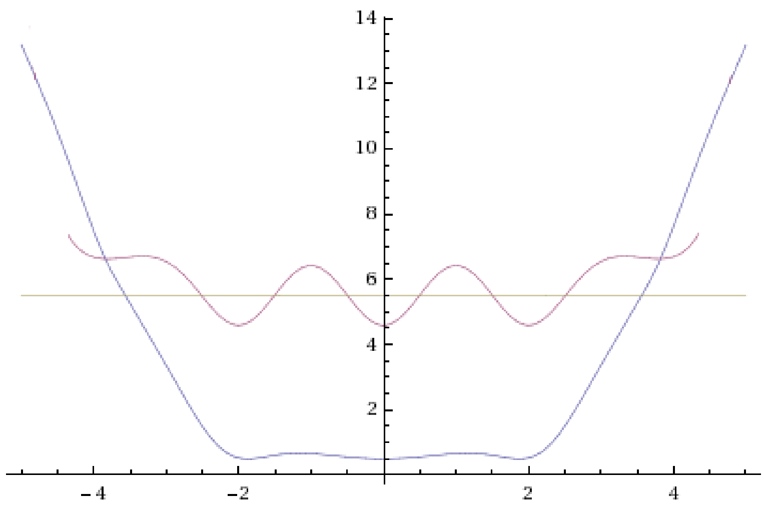
[Fig 2.2.4: $V_2(x), \psi_2^{(1)}(x)$ and $E_2^{(1)}$]



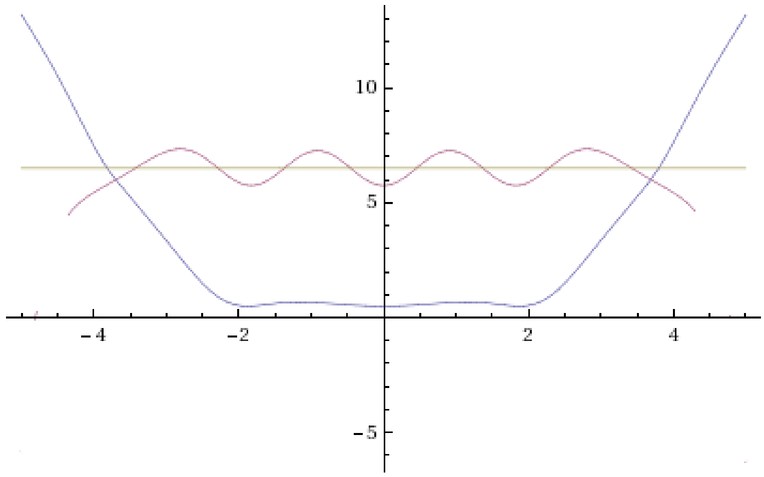
[Fig 2.2.5: $V_2(x), \psi_2^{(2)}(x)$ and $E_2^{(2)}$]



[Fig 2.2.6: $V_2(x), \psi_2^{(3)}(x)$ and $E_2^{(3)}$]

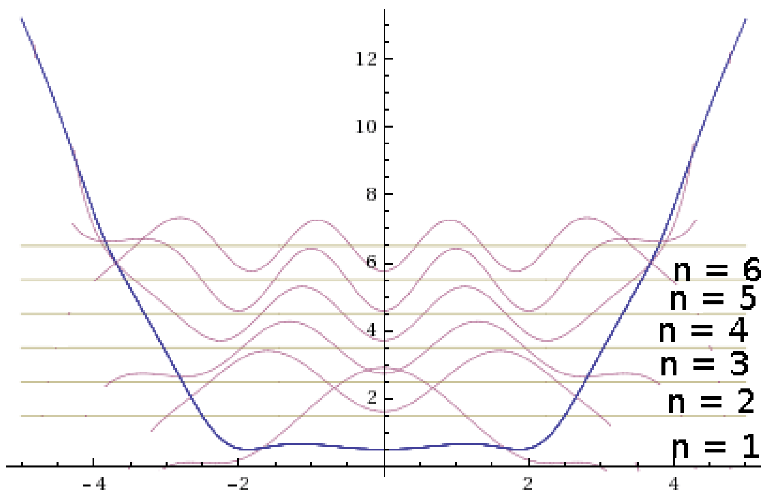


[Fig 2.2.7: $V_2(x), \psi_2^{(4)}(x)$ and $E_2^{(4)}$]



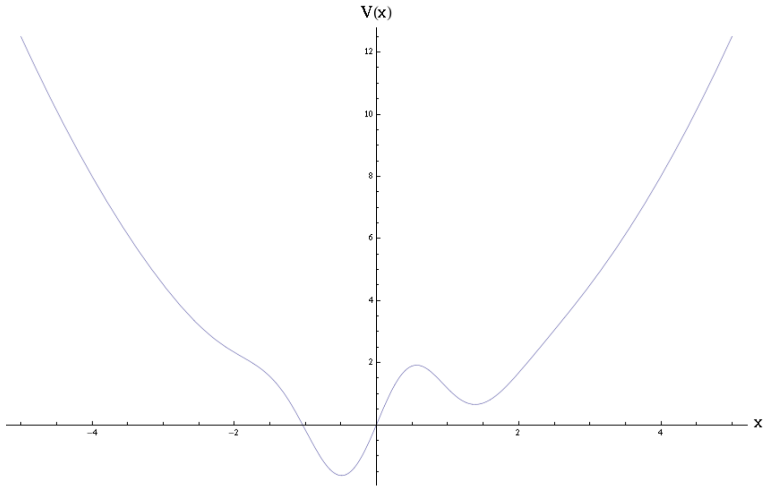
[Fig 2.2.8: $V_2(x), \psi_2^{(5)}(x)$ and $E_2^{(5)}$]

Wavefunctions for the potential V_2 are summarised as

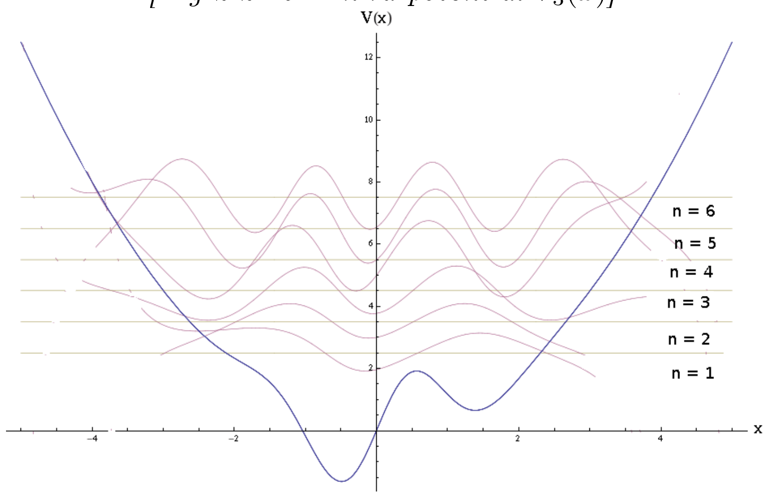


[Fig 2.2.9: The potential $V_2(x)$ with wavefuncitons corresponding eigenvalues]

We have calculated third superpartner using equation (2.2.21) analytically and got the potential V_3 and wavefunctions $\psi_3^{(m)}$ for V_3 as

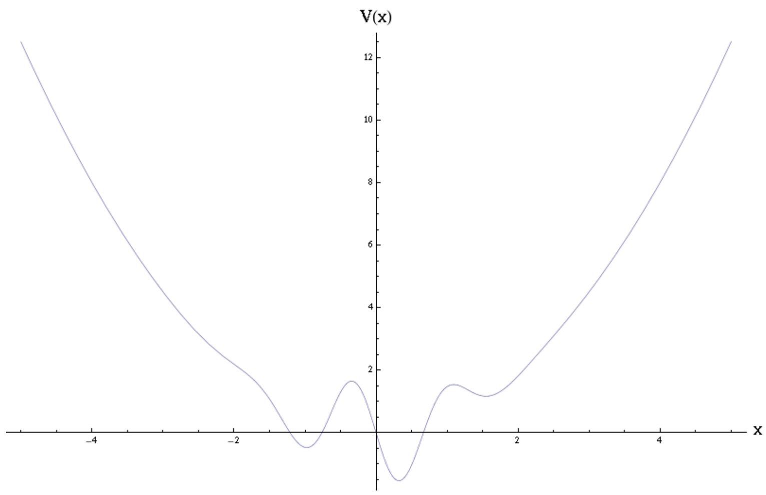


[Fig 2.2.10: Third potential $V_3(x)$]



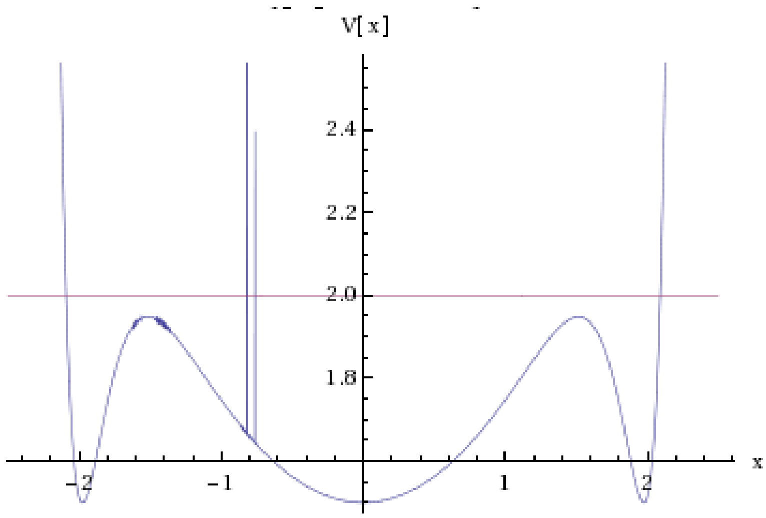
[Fig 2.2.11: The potential $V_3(x)$ with wavefuncitons corresponding eigenvalues]

Similarly, solving numerically we have found the following implementation of the potentials V_4 as

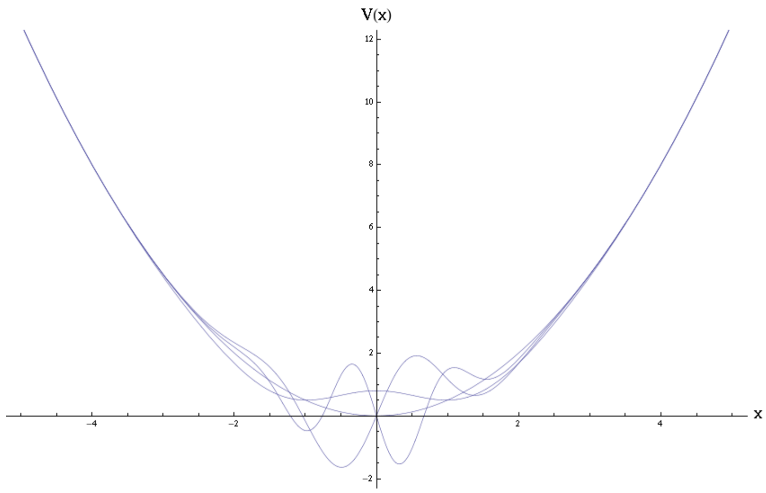


[Fig 2.2.12: The potential $V_4(x)$]

By solving numerically, we have found that this potential arise some singularity for V_4 as

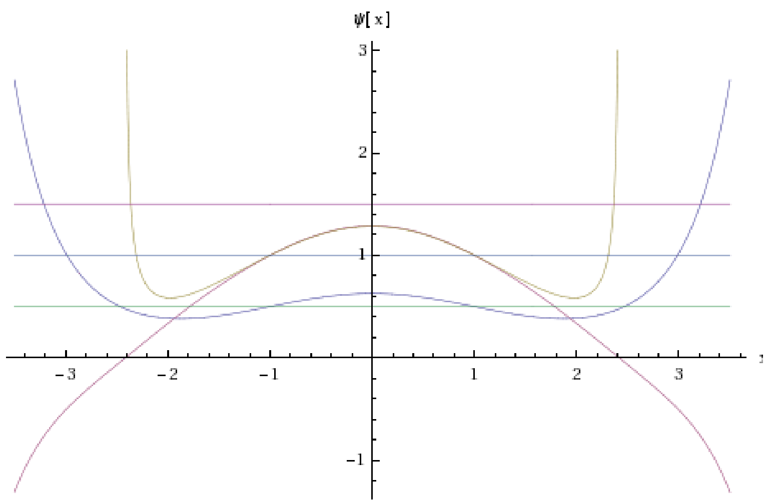


[Fig 2.2.13: $V_4(x), \psi_4^{(0)}(x)$ and $E_4^{(0)}$]



[Fig 2.2.14: Comparision of potential V_1, V_2, V_3 and V_4]

And the corresponding ground state wavefunction $\psi_1^{(0)}, \psi_2^{(0)}$ and $\psi_3^{(0)}$is found as



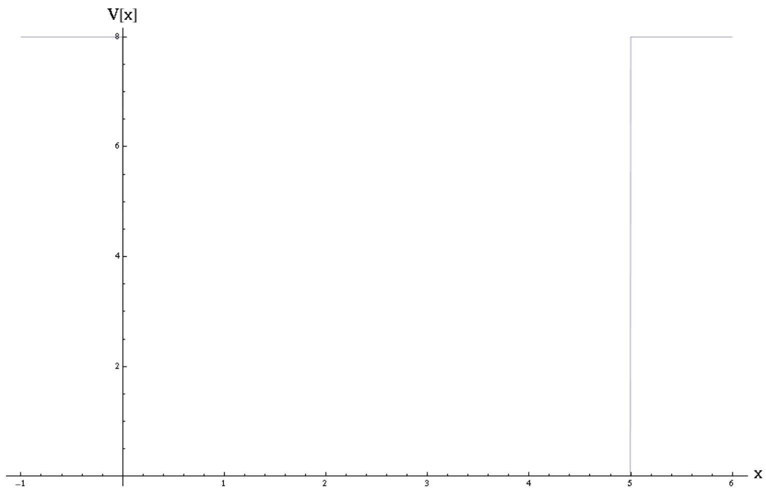
[Fig 2.2.15: Comparision of ground state wavefuncions $\psi_1^{(0)}, \psi_2^{(0)}$ and $\psi_3^{(0)}$]

3.3. Square Well Potential and Wavefunctions

Let us consider a finite square well potential such that

$$V(x) = \{ \begin{array}{ll} 0 & if\, 0 \leq x \leq a \\ V_0 & otherwise \end{array} \tag{3.3.1}$$

That is



[Fig 2.3.1: Square Well Potential]

There is a potential barrier at $x = 0$ and $x = a$. Let us consider that $V_0 = 8$ and $a = 5$.

At first we have to solve the Schroedinger Equation (SE). Since the potential inside the barrier is zero, we have the SE as

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2}\psi_1^{(m)}(x) = E_1^{(m)}\psi_1^{(m)}(x) \tag{3.3.2}$$

The analytical solution of this differential equation will be

$$\psi_1^{(m)}(x) = A\, sin(k^{(m)}x) + B\, cos(k^{(m)}x) \tag{3.3.3}$$

where A and B are arbitrary constant and

$$k^{(m)} = \sqrt{\frac{2\mu E_1^{(m)}}{\hbar^2}} \tag{3.3.4}$$

Putting the boundary condition as

$$\psi_1^{(m)}(0) = 0 \quad \& \quad \psi_1^{(m)}(a) = 0 \tag{3.3.5}$$

We have the solution as

$$\psi_1^{(m)}(x) = A\, sin(k^{(m)}x) \tag{3.3.6}$$

We can find out the constant A by taking the normalization condition as

$$\int_0^a \psi_1^{(m)*}\psi_1^{(m)} = 1 \tag{3.3.7}$$

which provide us

$$A = \sqrt{\frac{2}{a}} \tag{3.3.8}$$

Thus the wavefunction for the square well potential become

$$\psi_1^{(m)}(x) = \sqrt{\frac{2}{a}}\, sin(k^{(m)}x) \tag{3.3.9}$$

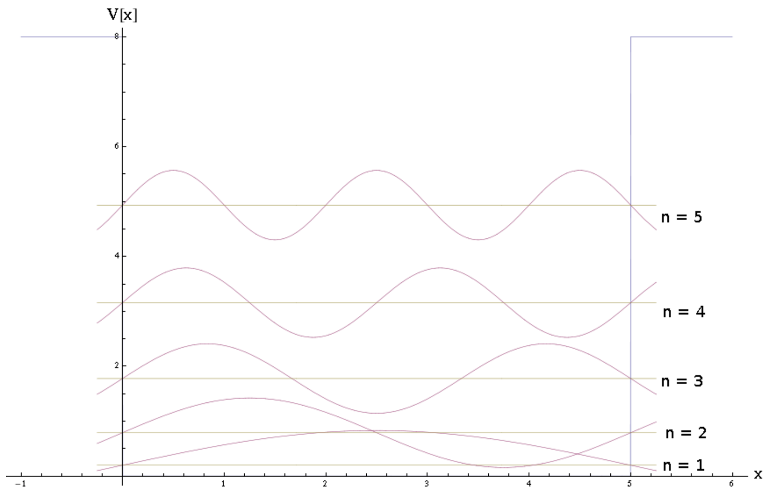
Since the wavefunction become zero at the boundary we must have

$$k^{(m)} = 0$$

which gives us the eigenvalue of energy as

$$E_1^{(m)} = \frac{\hbar^2}{2\mu}\frac{n^2\pi^2}{a^2} = n^2 E_1^{(1)} \qquad n = 1, 2, 3.......... \tag{3.3.10}$$

where the superscript 1 at right hand side denotes that the system is in ground state. With these eigenvalues of energy we have the wavefunction of a particle inside the potential well as plotted below.



[Fig 2.3.2: Square Well Potential and Wavefunctions with corresponding eigenvalues]

3.4. Supersymmetric Partner of Square Well Potential

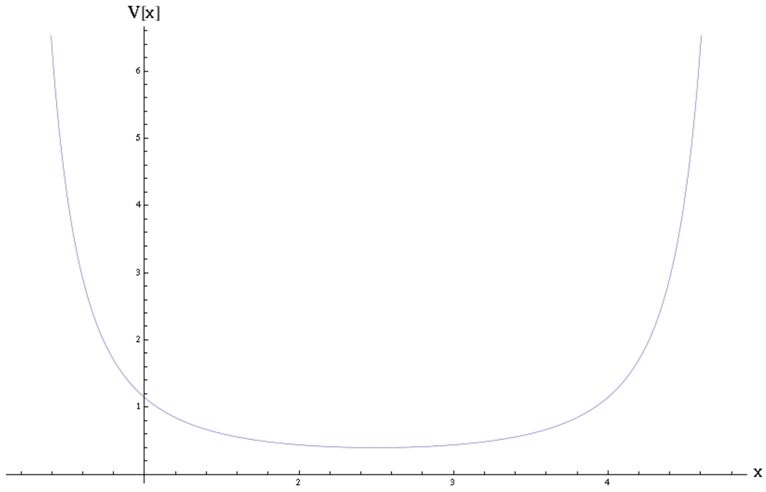
In this case, if we apply the Sukumar's process to find out the potential of the supersymmetric partner i.e.,

$$V_2(x) = V_1(x) - \frac{d^2}{dx^2} ln(\psi_1^{(1)}(x)) \tag{3.4.1}$$

Then we will get potential V_2 as

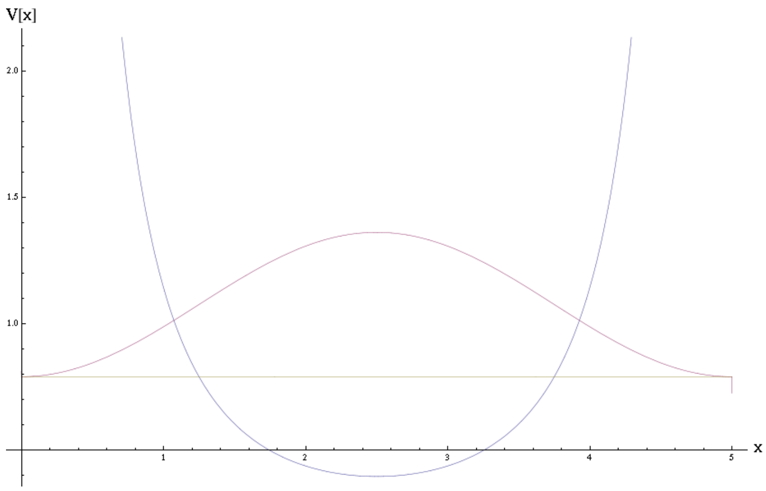
$$V_2(x) = V_1(x) + k^{(1)} cosec^2(k^{(1)}x) \tag{3.4.2}$$

Which gives the plot as



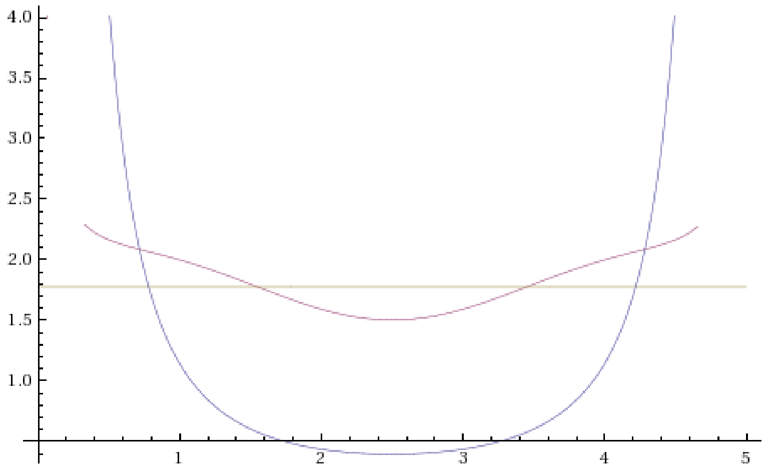
[Fig 2.4.1: V_2 for Squre Well Potential]

Eliminating the ground state eigenvalue, we have found the wavefunction of ground state of the superpartner as

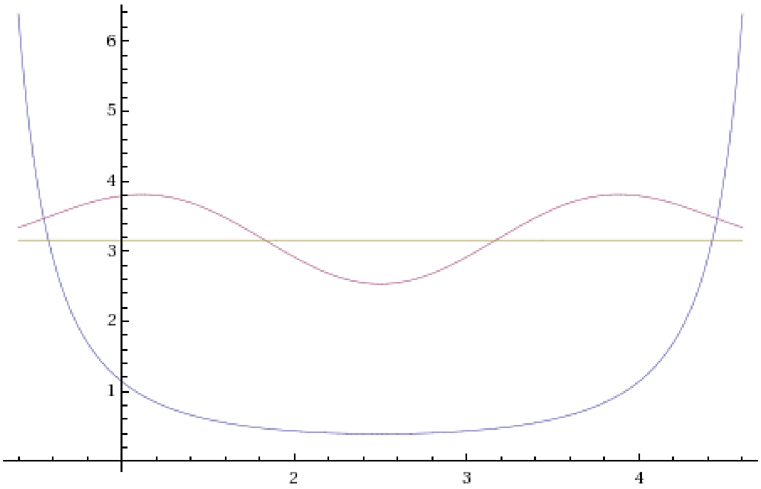


[Fig 2.4.2: $V_3(x), \psi_3^{(0)}(x)$ and $E_3^{(0)}$]

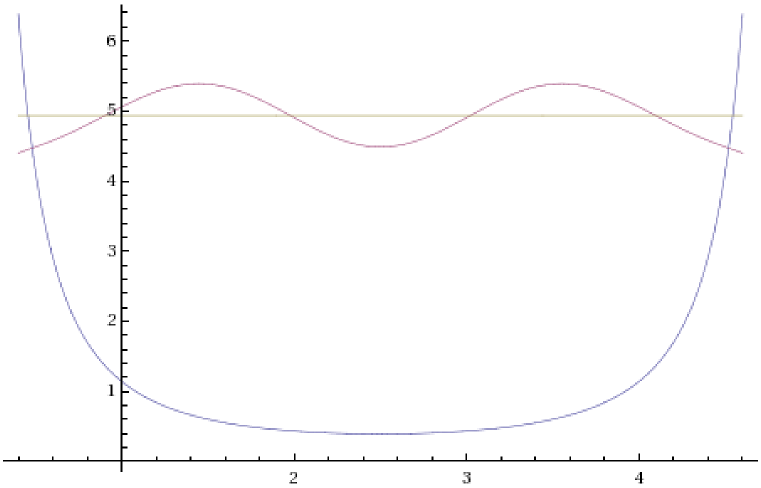
Similarly we have found the excited states as



[Fig 2.4.3: $V_3(x), \psi_3^{(1)}(x)$ and $E_3^{(1)}$]

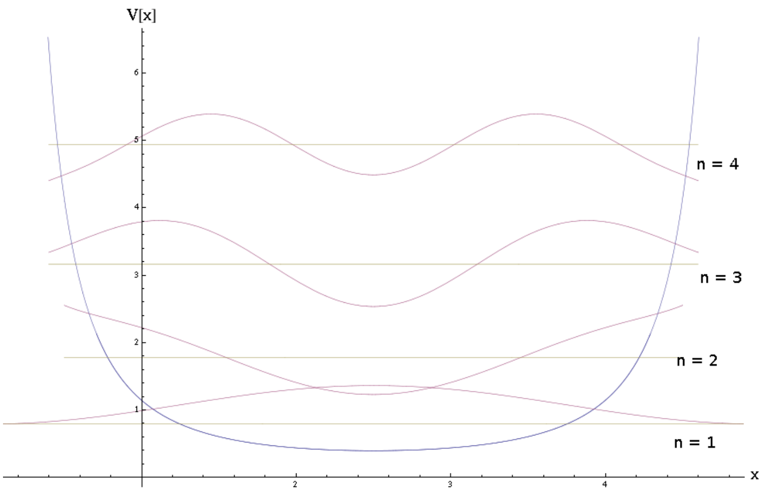


[Fig 2.4.4: $V_3(x), \psi_3^{(2)}(x)$ and $E_3^{(2)}$]



[Fig 2.4.5: $V_3(x), \psi_3^{(3)}(x)$ and $E_3^{(3)}$]

Thus we get the wavefunctions in the supersymmetric potential as



[Fig 2.4.6: Superpartner $V_2(x)$ for Square Well and it's wavefuncitons]

CHAPTER 4

RESULTS AND DISCUSSION

Since the factorization of the Hamiltonian is necessery to develope the hierarchy of the potential, we have done such factorization as was done by C. V. Sukumar [6]. Here we have found eigenvalue spectrum for different supersymmetric potential. We have considered the one dimensional Simple Harmonic Oscillator Potential (simple case) to calculate superpartner and wavefunctions with corresponding eigenvalues. In this case, we have calculated the wavefunctions for the SHO at first which is shown in Fig. 2.1.2. The remarkable option is that the ground state eigenvalue is not zero but $\frac{1}{2}$. Then we have calculated the first superpartner V_2, which is given by the Eq. 3.2.4. We have plotted the potential V_1 and V_2 in Fig. 2.2.2 to compare them. We have found the potential V_2 becomes two well in small value of x, which is predicted by M. Bernstein and L. S. Brown [3]. And there is a phase equivalnet for the higher value of x. Since we are going through the theory of elimination of ground state, we have taken the eigenvalue of first excited state of SHO potential as the ground state of its first superpartner. To do the analytical calculation with the potential V_2 becomes very tough. So, we have done numerical calculation here. Using the boundary condition $\psi_2(-1) = E_2$ and $\psi_2(1) = E_2$, we have calculated the wavefunctions from ground state to some extend, which is shown in Fig. 2.2.3 to Fig 2.2.8. Drawing these wavefunctions, we have taken the normalization condition such as we have just divided the wavefunctions by the amplitude found in the numerical calculations. We have summuraised all the wavefunctions for the potential V_2, which is shown in Fig 2.2.9, and have found that except ground state all the wavefunctions are similar with the wavefunctions for the potential V_1. After doing the calculations for potential V_2, we have moved towards calculating the potential V_3 analytically and we have found the plot of V_3 as shown in Fig 2.2.10. Again, we have used numerical calculations for the potential V_3 to find the wavefunctions by eliminating the ground state from the potential V_2. We have plotted the potential V_3 and wavefunctions with corresponding eigenvalues as shown in Fig 2.2.11. Again we have found the similar wavefuncitons except ground state. Then we have calculated the 4^{th} superpartner of the SHO potential V_4 which is shown in Fig 2.2.12. When we look at small value of this potential, we have found some singular point in this potential, which is shown in Fig 2.2.13. To compare all the potentials we have already calculated that are plotted the potentials in Fig 2.2.14. We have found that all the potential have phase equivalent in the high value. The ground state wavefunctions for the potential V_1, V_2 and V_3 are shown in Fig 2.2.15.

Then we have considered another potential called finite square well potential. We have taken the potential as given by the Eq. 3.3.1. We have considered the value of $V_0 = 8$ and the width of the well is given as $0 \leq x \leq 5$. Since the potential is zero inside the well, we have found the Schr$\ddot{o}$dinger equation as Eq 3.3.2. By solving this equation with corresponding eigenvalue and normalizing the wavefunctions we have found the wavefunctions which are shown in Fig 2.3.2. Then we have calculated the superpartner V_2 analytically as in Eq 3.4.2 and shown in Fig 2.4.1. By eliminating ground state, we have calculated the wavefunctions for the potential V_2 as shown in Fig. 2.4.2 to 2.4.5 which are summarized in Fig 2.4.6.

CHAPTER 5

CONCLUSION

In this paper, we have evaluated the potential of the supersymmetric partner in either case of Simple Harmonic Oscillator potential and Square well potential by the process of elimination of ground state. We have also plotted the wavefunction with the corresponding eigenvalue which is done by both analytical and numerical process. Then we have compared the potential and their superpartner and also wavefunctions in either case. Since this is a review work, we will illustrate more about the Hamiltonian hierarchy in thesis work.